

Testing Dark Energy Models through Large Scale Structure

Olga Avsajanishvili¹, Lado Samushia^{2,1}, Natalia A. Arkhipova³, and Tina Kahniashvili^{4,5,1}

¹ Abastumani Astrophysical Observatory, Ilia State University, 3-5 Cholokashvili Ave., Tbilisi, 0194, Georgia

² Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS, 66506, USA

³ Astro Space Center of P. N. Lebedev Physical Institute, Russia, 84/32 Profsoyuznaya str., Moscow, 117997 Russia

⁴ McWilliams Center for Cosmology and Department of Physics, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213 USA

⁵ Department of Physics, Laurentian, University, Ramsey Lake Road, Sudbury, ON P3E 2C, Canada

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ABSTRACT

Context. This paper is a contribution to the Proceedings of the 5th Gamow International Conference, Odessa, Ukraine, August 2015.

We explore the scalar field quintessence freezing model of dark energy with the inverse Ratra-Peebles potential. We study the cosmic expansion and the large scale structure growth rate. We use recent measurements of the growth rate and the baryon acoustic oscillation peak positions to constrain the matter density Ω_m parameter and the model parameter α that describes the steepness of the scalar field potential.

Aims. To study the background dynamics of the ϕ CDM model. To investigate the influence of the scalar field on the expansion rate of the Universe. To examine the background evolution of the ϕ CDM model on the cosmological model parameters and on the content of the Universe. To study the equation of state parameter $w(a)$ (with the scale factor a) during the expansion of the Universe. To investigate the influence of the ϕ CDM model on evolution of the large scale structure. To explore the applicability of the Linder γ -parametrization of growth rate for the ϕ CDM model and to define a redshift range of this parametrization validity. To derive observational constraints on the model parameters Ω_m and α .

Methods. We solve jointly the equations for the background expansion and for the growth rate of matter perturbations. The obtained theoretical results are compared with the observational data. We perform the Bayesian data analysis to derive constraints on the model parameters.

Results. The larger value of the α parameter implies stronger time dependence of the scalar field. For the Ratra-Peebles ϕ CDM model the expansion of the Universe occurs faster with increasing of the α parameter. The scalar field begins to be dominant earlier with the increasing value of the α parameter. The Ratra-Peebles ϕ CDM model predicts a slower growth rate than the Λ CDM model. The Linder γ -parametrization works well for the Ratra-Peebles ϕ CDM model in the range of the redshifts $z \in (0; 5)$.

Key words. dark energy, scalar field, expansion rate, growth rate, growth index, large scale structure

1. Introduction

According to the cosmological observations our Universe expands with an acceleration (Perlmutter et al., 1999; Riess et al., 1998, 2007). There are several models for explanation of this phenomenon. The most common approach is to assume that order of 70% of the energy density of the Universe is present in the form of dark energy (DE).

The simplest model of DE admits that DE is the vacuum energy, that is given in the form of the time-independent cosmological constant Λ . This model is referred to as a *concordance* model since it is in a very good agreement with all available today cosmological observations. The Λ CDM model, however, suffers from the fine tuning and the coincidence problems (Carroll, 2001; Padmanabhan, 2003; Peebles & Ratra, 2003; Martin, 2012). To alleviate these problems, other models of DE have been proposed (Caldwell & Steinhardt, 1998;

Armendariz-Picon et al., 1999; Amendola, 2000; Wetterich, 1995; Kamenshchik et al., 2001; Capozziello et al., 2003; Dvali et al., 2000; Shi & Baugh, 2015; Duniya, 2015; Chen & Xu, 2016; Pradhan et al., 2015). The main alternatives of the Λ CDM model are the models involving a dynamical scalar field, so called the ϕ CDM models. This family of models avoids some theoretical foundation difficulties, namely: the fine tuning problem, having a more natural explanation for the observed low energy scale of DE (Zlatev et al., 1999). For the Λ CDM model the equation of state parameter w is a constant and is equal to minus one. For the ϕ CDM models the equation of state w parameter is time dependent (we will use below the scale factor function $w(a)$) and approaches to minus one today, i.e. $w(a_0) \rightarrow -1$, here a_0 is the today value of the scale factor normalized to be one, $a_0 \equiv a_{\text{today}} = 1$ (Caldwell & Linder, 2005; Yoo & Watanabe, 2012).

Depending on the value of the equation of state w parameter today, the ϕ CDM models are divided into two classes: the phantom models ($-1/3 < w < -1$) and the quintessence models ($w > -1$). The quintessence models are subdivided into the thawing models, for which the evolution of the scalar field is fast (Scherrer & Sen, 2008; Linder, 2015; Lima et al., 2015) and the tracking (freezing) models, for which the evolution of the scalar field is slow, compared to the Hubble expansion (Caldwell et al., 1998; Caldwell & Linder, 2005; La Vacca & Kristiansen, 2009; de Putter & Linder, 2008; Chiba et al., 2013). In the tracking models, the scalar field has a tracking solution, in which the scalar field energy density, remaining subdominant, tracks at first the radiation and then the matter energy densities (Brax & Martin, 2002; Steinhardt et al., 1999). At late times, the scalar field becomes the dominant component and starts to behave as a component with the effective negative pressure, that leads at the late stages to the accelerated expansion of the Universe. The simplest representation of such a model is the scalar field model, when the scalar field potential is given through an inverse power law, Ratra-Peebles, potential (Ratra & Peebles, 1988). We will refer to this model below as the ϕ CDM Ratra-Peebles model.

The background expansion history (as well as the large scale structure growth rate) is different for the scalar field (ϕ CDM) model and for the concordance (Λ CDM) model. Thus the ϕ CDM model can be distinguished from the Λ CDM model through high precision measurements of distances and growth rates over a wide redshift range (Samushia et al., 2012; di Porto et al., 2012; Pavlov et al., 2012; Bueno Belloso et al., 2011; Fu et al., 2010; Lee & Ng, 2010; Pace et al., 2012; Ahn et al., 2009; de Putter & Linder, 2008; Silvestri & Trodden, 2009).

We present solutions of the joint equations for the background expansion and for the growth rate of matter perturbations. We use a compilation of the recent measurements of the growth rate and the baryon acoustic oscillation (BAO) to put constraints on the α and the Ω_m parameters.

This paper is organized as follows. In Sec. 2 we investigate the dynamics and the energy of the Ratra-Peebles ϕ CDM models. In Sec. 3 we explore the parametrization of the equation of state parameter $w(a)$ in the Ratra-Peebles ϕ CDM model by the different models. In Sec. 4 we study the influence of the Ratra-Peebles ϕ CDM models on the growth rate. In Sec. 5 we present the comparison of the obtained theoretical results with the observational data. We discuss our results and conclude in Sec. 6. We use the natural units with $c = \hbar = 1$ throughout this paper.

2. Background dynamics in the Ratra-Peebles ϕ CDM model

2.1. Background equations

We study the ϕ CDM model with the Ratra-Peebles potential given by (Ratra & Peebles, 1988):

$$V(\phi) = \frac{\kappa}{2} M_{\text{pl}}^2 \phi^{-\alpha}, \quad (1)$$

where M_{pl} is the Planck mass, κ is a model parameter, and α is a positive constant, which determines the steepness of the scalar field potential. Current observational data suggest that α can not be larger than $\alpha \leq 0.7$ (Samushia, 2009;

Pavlov et al., 2012; Farooq & Ratra, 2013). A larger value of the α parameter implies a stronger time dependence of the scalar field potential $V(\phi)$. In the limit of $\alpha=0$, the Ratra-Peebles ϕ CDM model reduces to the Λ CDM model.

The equation of motion for the scalar field is (Ratra & Peebles, 1988; Sahni, 2002):

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{2}\kappa\alpha M_{\text{pl}}^2 \phi^{-(\alpha+1)} = 0, \quad (2)$$

where an over-dot represents the derivative with respect to a physical time t , $H(a) = \dot{a}/a$ is the Hubble parameter, the scale factor is $a = 1/(1+z)$, and z is a redshift.

The energy density and the pressure of the scalar field are (Ratra & Peebles, 1988; Sahni, 2002)

$$\rho_\phi = \frac{M_{\text{pl}}^2}{64\pi} (\dot{\phi}^2 + \kappa M_{\text{pl}}^2 \phi^{-\alpha}), \quad (3)$$

$$P_\phi = \frac{M_{\text{pl}}^2}{64\pi} (\dot{\phi}^2 - \kappa M_{\text{pl}}^2 \phi^{-\alpha}), \quad (4)$$

and, the corresponding equation of state w parameter is,

$$w = \frac{\dot{\phi}^2 - \kappa M_{\text{pl}}^2 \phi^{-\alpha}}{\dot{\phi}^2 + \kappa M_{\text{pl}}^2 \phi^{-\alpha}}. \quad (5)$$

The scalar field energy density parameter is defined by,

$$\Omega_\phi(a) = \frac{1}{12H_0^2} (\dot{\phi}^2 + \kappa M_{\text{pl}}^2 \phi^{-\alpha}). \quad (6)$$

and the first Friedmann equation for the Ratra-Peebles ϕ CDM model in spatially-flat Universe is:

$$E^2(a) = \Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \frac{1}{12H_0^2} (\dot{\phi}^2 + \kappa M_{\text{pl}}^2 \phi^{-\alpha}), \quad (7)$$

where $E(a) = H(a)/H_0$, with $H_0 = 100h\text{km/s/Mpc}$ is a value of Hubble parameter today; Ω_{r0} , Ω_{m0} , and $\Omega_{\phi0}$ the dimensionless density parameters for radiation, matter and DE at present time. During the late stages of the Universe's expansion (after radiation-matter equality) we can neglect the radiation term in the Eq. (7). In what follows we will assume fiducial values $\Omega_{m0} = 0.315$, $\Omega_{\phi0} = 0.685$, $h = 0.673$ consistent with (Ade et al., 2014).

2.2. Initial conditions

We integrate the set of equations Eq. (2), Eq. (6), and Eq. (7) numerically, starting from $a_{\text{in}} = 5 \cdot 10^{-5}$ to the present time $a_0 = 1$. We assume the following initial conditions for the scalar field amplitude and its time derivative,

$$\phi_{\text{in}} = \left[\frac{1}{2}\alpha(\alpha+2) \right]^{1/2} a_{\text{in}}^{\frac{4}{\alpha+2}}, \quad (8)$$

$$\phi'_{\text{in}} = \left(\frac{2\alpha}{\alpha+2} \right)^{1/2} a_{\text{in}}^{\frac{2-\alpha}{2+\alpha}}, \quad (9)$$

$$\kappa = \left(\frac{\alpha+6}{\alpha+2} \right) \left[\frac{1}{2}\alpha(\alpha+2) \right]^{\alpha/2}, \quad (10)$$

where a prime denotes the differentiation with respect to scale factor a . To obtain these initial conditions we use the fact that the expansion of the Universe during matter and

radiation domination epochs has a power-law form and use the ansatz

$$a(t) = a_* \left(\frac{t}{t_*} \right)^n, \quad \phi(t) = \phi_* \left(\frac{t}{t_*} \right)^p \quad (11)$$

where $a_* \equiv a(t_*)$ and $\phi_* \equiv \phi(t_*)$ are the scale factor and the scalar field values at $t = t_*$. The index n depends on the dominant component driving the expansion of the Universe and is $n = 1/2$ in radiation dominated epoch and $n = 2/3$ in matter dominated epoch. We solve the set of equations Eq. (2), Eq. (6), and Eq. (7), during the radiation (and/or the matter dominated) epochs, to obtain the general expressions for κ , and the scalar field amplitude ϕ , and its time-derivative $\dot{\phi}$ (which depend only on the α parameter and value of the index n ; the details are given in Appendix A of (Avsajanishvili et al., 2014)):

$$\phi = n\alpha(\alpha + 2)^{1/2} \left(\frac{a}{a_*} \right)^{2/n(\alpha+2)}, \quad (12)$$

$$\kappa = \frac{4n}{M_{\text{pl}}^2 t_*^2} \left(\frac{6n + 3n\alpha - \alpha}{\alpha + 2} \right) [n\alpha(\alpha + 2)]^{\alpha/2}, \quad (13)$$

We set $t_* = M_{\text{pl}}^{-1}$ and obtain Eq. (8), Eq. (9), and Eq. (10) assuming the initial conditions are set in radiation dominated epoch ($n = 1/2$).

2.3. The dynamics and the energy of the Ratra-Peebles ϕ CDM model.

In this subsection we examine the evolution of the equation of state parameter $w(a)$ and its scale factor derivative $w'(a)$ for different values of the α parameter (see Fig.1). The equation of state parameter $w(a)$ is a decreasing function of time (with the increasing scale factor): that is a specific feature of the freezing models. In fact, a large value of the α parameter results in a stronger time dependence for the equation of state parameter $w(a)$ (see Fig.1a) and its scale factor derivatives $w'(a)$ (see Fig.1b).

Next, we investigate the influence of the scalar field ϕ on the expansion rate of the Universe. For the Ratra-Peebles ϕ CDM model this expansion occurs faster with increasing value of the α parameter (see Fig. 2a). The Λ CDM limit corresponds to the slowest rate of the Universe's expansion.

We also study the background dynamics (see Fig. 3a), and the evolution of the energy density of the matter and DE components, $\Omega_m(a)$ and $\Omega_\phi(a)$ respectively (see Fig. 3b) for the Ratra-Peebles ϕ CDM model. As we can see in the Ratra-Peebles ϕ CDM model DE begins to be a dominant component earlier than in the Λ CDM model, and the effect is stronger for the larger value of the α parameter (see Fig. 3b), and thus the duration of the matter dominated epoch becomes shorter.

3. Parametrization of the equation of state parameter $w(a)$ in the Ratra-Peebles ϕ CDM model

There are several ways to parameterize the equation of state parameter $w(a)$, including:

- Cooray and Huterer (CH) (Cooray & Huterer, 1999)
 $w(a) = w_0 + w_a z$,

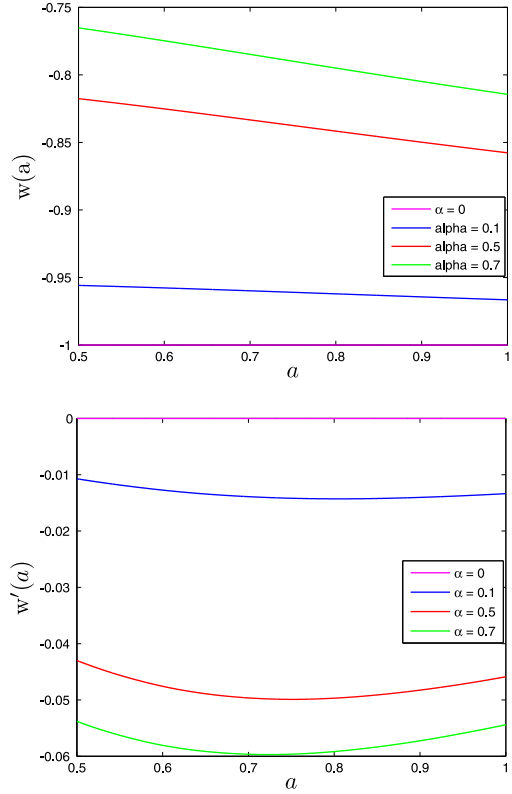


Fig. 1. 1a (upper panel) the DE equation of the equation of state parameter $w(a)$ for the different values of α parameter and 1b (lower panel) the scale factor derivative $w'(a)$ of the equation of state parameter for the different values of the α parameter.

- Gerke and Estathiou (GE) parametrization (Efstathiou, 1999)
 $w(a) = w_0 + w_a \ln(1 + z)$,
- Chavalier and Polarsky, and Linder (CPL) parametrization (Chevallier & Polarski, 2001; Linder, 2003)
 $w(a) = w_0 + w_a(1 - a)$,
- Barboza and Alcaniz (BA) parametrization (Barboza & Alcaniz, 2008)
 $w(a) = w_0 + w_a \frac{z(1+z)}{1+z^2}$,

where w_0 corresponds to the present day of the equation of state parameter $w(a)$, and $w_a = (dw/dz)|_{z=0} = (-dw/da)|_{a=1}$.

The parametrization of the equation of state parameter $w(a)$ is used as a method to distinguish the different DE models among themselves (Scherrer, 2015). In particular, this approach can be used to distinguish the Λ CDM and the ϕ CDM models at present moment. We present our results on the Fig. 4 - Fig. 5. BA and CH parameterizations fit better the equation of state parameter $w(a)$ in the Ratra-Peebles ϕ CDM model in the range of the redshifts $z \in (0; 1)$ then GE and CPL parameterizations. The CH, GE, CPL, BA $w(a)$ parameterizations and the function $w(a)$ in the Ratra-Peebles ϕ CDM model, for $\alpha = 0.7$ in the range of the redshifts $z \in (0; 1)$ are represented on the Fig. 6a. But for the early epochs (large redshifts), the CPL and BA $w(a)$ parameterizations approximate better the equation of state

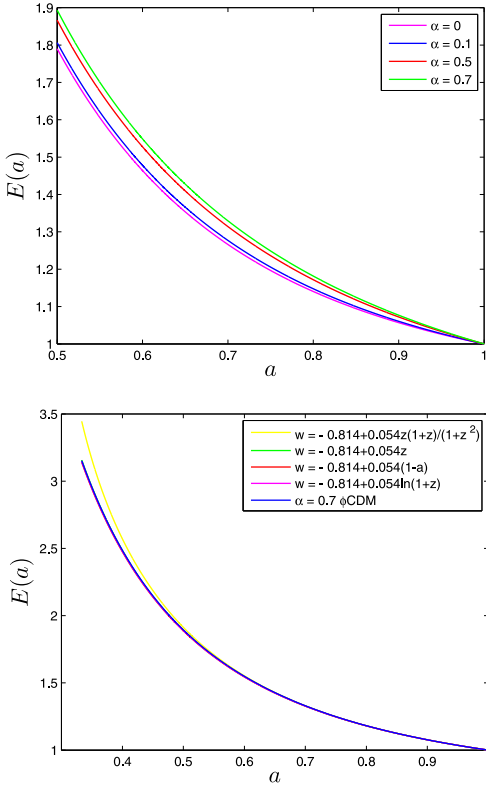


Fig. 2. 2a (upper panel) the normalized Hubble expansion rate $E(a)$ for the Ratra-Peebles ϕ CDM model for different values of the α parameter. 2b (lower panel) the normalized Hubble expansion rate $E(a)$ for BA, CH, GE, CPL parameterizations of $w(a)$ and for the Ratra-Peebles ϕ CDM model, $\alpha = 0.7$.

parameter $w(a)$ in the Ratra-Peebles ϕ CDM model for the large redshifts then BA and CH parameterizations (see the Fig. 6b).

We evaluate the Universe expansion rate for the BA, CH, GE and CPL parameterizations (Carroll, 2004):

$$E^2(a) = \Omega_{m0}a^{-3} + \Omega_{\phi0}e^{-3 \int_1^a \frac{da}{a}(1+w(a))}, \quad (14)$$

and we compare them with the value of the expansion rate for the Ratra-Peebles ϕ CDM model. The expansion rates for all aforementioned parameterizations of $w(a)$ fit well the expansion rate for the Ratra-Peebles ϕ CDM model for $\alpha = 0.7$ in the range of the redshifts $z \in (0; 0.6)$ (see Fig. 2b).

4. Growth factor of matter density perturbations in the Ratra-Peebles ϕ CDM model

In this section we investigate the influence of the scalar field (of the Ratra-Peebles ϕ CDM model) the growth of structure (Pavlov et al., 2014; Taddei & Amendola, 2015). We integrate numerically the linear perturbation equation Eq. (15) with respect to the relative density contrast $\delta \equiv \delta\rho_m/\bar{\rho}$, where $\bar{\rho}$ and $\delta\rho_m$ - are the mean density and overdensity of the matter component respectively. We apply the initial conditions: $\delta(a_{in}) = \delta'(a_{in}) = 5 \cdot 10^{-5}$ (Pace et al., 2010; Campanelli et al., 2012).

$$\delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' - \frac{3\Omega_{m0}}{2a^5E^2}\delta = 0, \quad (15)$$

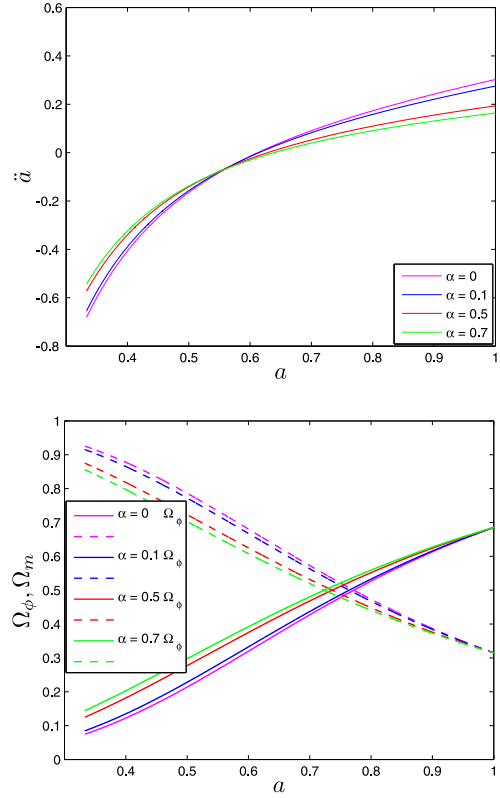


Fig. 3. 3a (upper panel) the second derivative of the scale factor and 3b (lower panel) the matter energy density $\Omega_m(a)$ the (dashed lines) and the scalar field energy density $\Omega_\phi(a)$ (solid lines) as functions of the scale factor for different values of the α parameter.

Eq. (15) describes completely the dynamical evolution of matter perturbations, assuming that the perturbed fluid is a perfect one. We study the evolution of the perturbations through the linear growth factor $D(a) = \frac{\delta(a)}{\delta(a_0)}$, where $\delta(a_0)$ - is a value of the density contrast today. We normalize the linear growth factor $D(a)$ to be unity today, i.e. $D(a_0) = 1$.

For the Ratra-Peebles ϕ CDM model a larger value of the α parameter implies a stronger time dependence of the linear growth factor $D(a)$, see the Fig. 7a. This is because the growth of matter perturbations occurs only during matter dominated epoch (Frieman et al., 2008). The Hubble expansion takes place faster for larger values of the α parameter (see the Fig. 2a), while the scalar field energy domination begins earlier, see the Fig. 3b. As a result, the matter perturbations have less time to grow. To reach the same amplitude of matter perturbation $\delta(a_0)$ today, the models where the scalar field has a larger value of α will require larger initial amplitudes.

The growth rate $f_2(a) = d \ln D(a) / d \ln a$ strongly depends on the fractional matter density $f_1(a) = \Omega_{m0}a^{-3}/E^2$ and this dependence can be parametrized by a power-law relationship (Wang & Steinhardt, 1998)

$$f_2(a) \approx [f_1(a)]^\gamma, \quad (16)$$

where γ is a growth index, the value of which depends on the model parameters. Assuming GR is correct, the value of γ -parameter depends on dark energy parameters

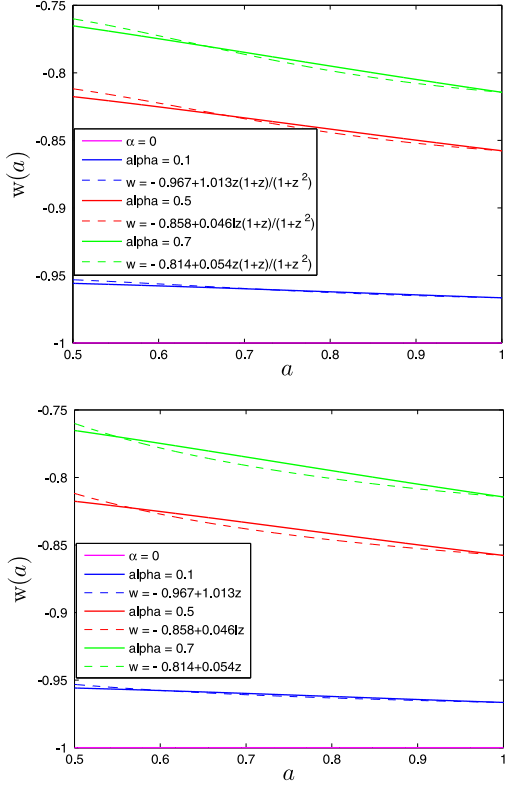


Fig. 4. 4a (upper panel) $w(a)$ for different values of the α parameter along with predictions computed for the BA parametrization with corresponding best-fit values for w_0 and for w_a . 4b (lower panel) $w(a)$ for different values of the α parameter along with predictions computed for CH parametrization with corresponding best-fit values for w_0 and for w_a .

as (Linder, 2005).

$$\gamma = 0.55 + 0.05(1 + w_0 + 0.5w_a), \quad \text{if } w_0 \geq -1. \quad (17)$$

For the Λ CDM model (with $w_a = 0$, $w_0 = -1$), the growth index $\gamma \approx 0.55$.

We study the applicability of the the power-law approximation Eqs. (16) for the Ratra-Peebles ϕ CDM model. This approximation works well for the Ratra-Peebles ϕ CDM model (see Fig. 7b). The value of the growth index γ for the ϕ CDM model depends on the α parameter. The value of the growth index γ for the ϕ CDM model increases with increasing value of the α parameter, and it is slightly higher than one for the Λ CDM model ($\gamma \approx 0.55$). The growth rate of matter perturbations occurs slower with the increasing value of the α parameter (see Fig. 7b). This results that the Hubble expansion and the growth of matter perturbations are interrelated and oppositely directed processes, and the faster Hubble expansion for the larger α parameter (see the Fig. 2a) leads to the suppression of the growth of matter perturbations.

We examine also the applicability of the Linder γ -parametrization on the large redshifts. This parametrization occurs in the range of the redshifts $z \in (0; 5)$ (see Fig. 9a) and for the larger values of the redshifts the Linder γ -parametrization is not applicable.

We consider also the evolution of the $\gamma(a)$ function with its dependence on the scale factor a (Wu et al., 2009) for

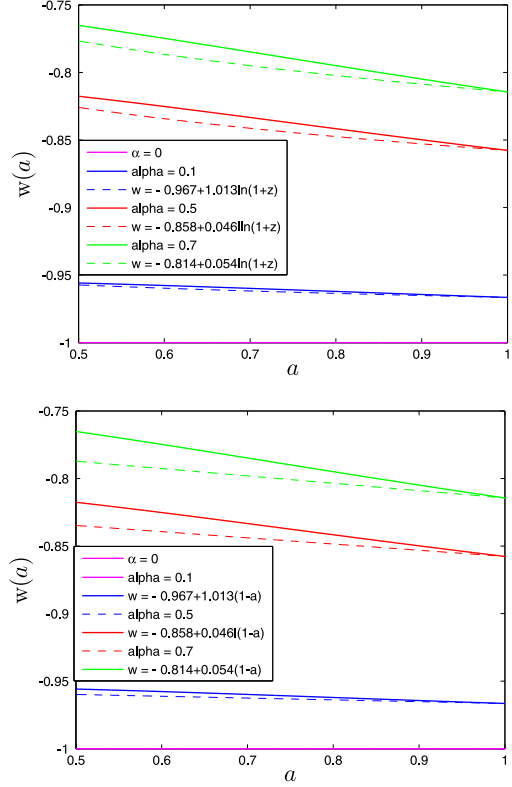


Fig. 5. 5a (upper panel) $w(a)$ for different values of the α parameter along with predictions computed for GE parametrization with corresponding best-fit values for w_0 and for $w(a)$. 5b (lower panel) $w(a)$ for different values of the α parameter along with predictions computed for the CPL parametrization with corresponding best-fit values for w_0 and for w_a .

the Ratra-Peebles ϕ CDM model. From Eqs. (16) we have (Linder & Cahn, 2007; Basilakos, 2015; Mehrabi et al., 2015):

$$\gamma(a) = \frac{\ln f_2(a)}{\ln f_1(a)}.$$

The $\gamma(a)$ functions in the range of the redshifts $z \in (0; 1)$ for different values of the α parameters are represented on the Fig. 8a. We approximate the $\gamma(a)$ function for different values of the α parameters by the linear functions (see Fig. 8b).

Next, we investigate the behavior of the $\gamma(a)$ function on the large redshifts, see Fig. 9b. The linear dependence of the $\gamma(a)$ function breaks off in the range of the redshifts from $z \approx 3$ for $\alpha = 0$ till $z \approx 5$ for $\alpha = 0.7$. Thus the linear dependence of the $\gamma(a)$ function breaks off earlier for the Λ CDM model then for the ϕ CDM models. Comparing the Fig. 9a and the Fig. 9b we see, that the termination of the applicability of the Linder γ -parametrization for different values of the α parameter coincides with the moments of the termination of the linear dependence of the $\gamma(a)$ function.

5. Comparison with observations

In this section we carry out the observational constraints on the α and the Ω_m parameters using χ^2 analysis, where calculated values of the growth rates are compared with

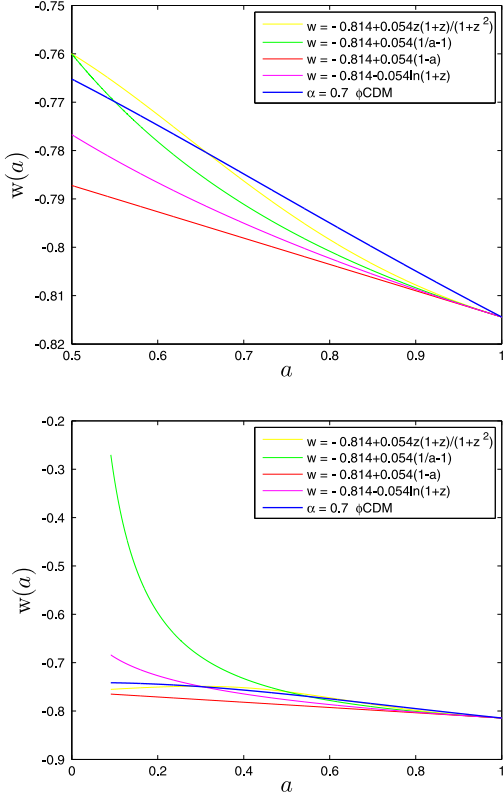


Fig. 6. 6a (upper panel) CH, GE, CPL, and BA parameterizations of the $w(a)$ and $w(a)$ for the Ratra-Peebles ϕ CDM model, $\alpha = 0.7$ in the range of the redshifts $z \in (0; 1)$. 6b (lower panel) BA, CH, GE, CPL parameterizations of the $w(a)$ and $w(a)$ for the Ratra-Peebles ϕ CDM model, $\alpha = 0.7$ in the range of the redshifts $z \in (0; 5)$.

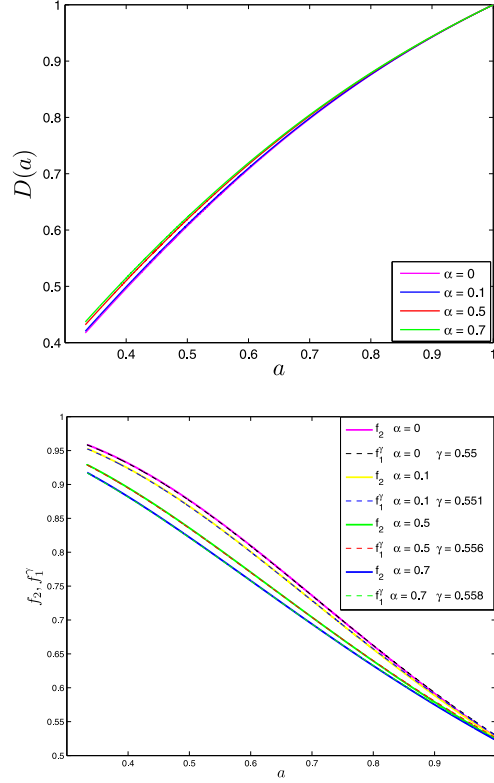


Fig. 7. 7a (upper panel) the linear growth $D(a)$ as a function of the scale factor for different values of the α parameter. 7b (lower panel) the logarithmic growth rate as a function of the scale factor f_2 for different values of the α parameter (solid lines) along with the predictions f_1^γ (dashed lines), computed for the corresponding best-fit values of the γ parameter.

the observational ones, obtained from the redshift space distortion surveys. For this purpose we use a compilation of the growth rate measurements from (Gupta et al., 2012).

The 1 and 2σ confidence contours resulting from this likelihood are presented on the Fig. 10a. The likelihood contours in the $\alpha - \Omega_m$ plane obtained from the growth rate data alone are highly degenerate. If we fix $\alpha = 0$ we get $\Omega_m = 0.278 \pm 0.03$ which is within 1σ of the best-fit value obtained by Planck collaboration (Ade et al., 2014). To break the degeneracy between the Ω_m and the α parameters we carry out the χ^2 BAO/CMB analysis (Giostri et al., 2012). Where we construct the ratio of the angular distance d_A and the distance scale D_V :

$$\eta(z) \equiv d_A(z_{\text{bao}})/D_V(z_{\text{bao}}),$$

Assuming Gaussianity of the errorbars we compute

$$\chi_{\text{bao}}^2 = \mathbf{X}^T \mathbf{C}^{-1} \mathbf{X}, \quad (18)$$

and a likelihood function

$$\mathcal{L}^{\text{bao}}(\alpha, \Omega_m, H_0) \propto \exp(-\chi_{\text{bao}}^2/2), \quad (19)$$

where $\mathbf{X} = \eta_{\text{th}} - \eta_{\text{m}}$ and \mathbf{C} is the covariance matrix of the measurements. To marginalize over parameter H_0 in \mathcal{L}^{bao} we take a Gaussian prior of $H_0 = 74.3 \pm 2.1$ from (Freedman et al., 2012). We assume that \mathcal{L}^{f} and \mathcal{L}^{bao} are

independent and the combined likelihood is simply a product of the two. The results are presented on the Fig.10b.

As a result of the BAO/CMB analysis we've obtained the restrictions on the parameters Ω_m and α . Ω_m is now constrained to be within $0.26 < \Omega_m < 0.34$ at 1σ confidence level. For α parameter we get $0 \leq \alpha \leq 1.3$ at 1σ confidence level.

6. Discussion and Conclusions

Analyzing the obtained results we can conclude that the Ratra-Peebles ϕ CDM model differs from the Λ CDM model in number of ways. These distinctive features are generic and do not depend on the specific values of model parameters. In the Ratra-Peebles ϕ CDM model the expansion rate of the Universe is always faster than for the Λ CDM model. The DE dominated epoch sets in earlier than for the Λ CDM model. The scalar field model predicts a slower growth rate than the Λ CDM model.

Below we summarize our results:

We've investigated the parametrization of the equation of state parameter $w(a)$ for the Ratra-Peebles ϕ CDM model by the Coorkay-Huterer, Gerke-Estathiou, Chavalier-Polarsky-Linder, and Barboza-Alcaniz models. We've found that the Barboza-Alcaniz and Coorkay-Huterer parameterizations fit well the equation of state

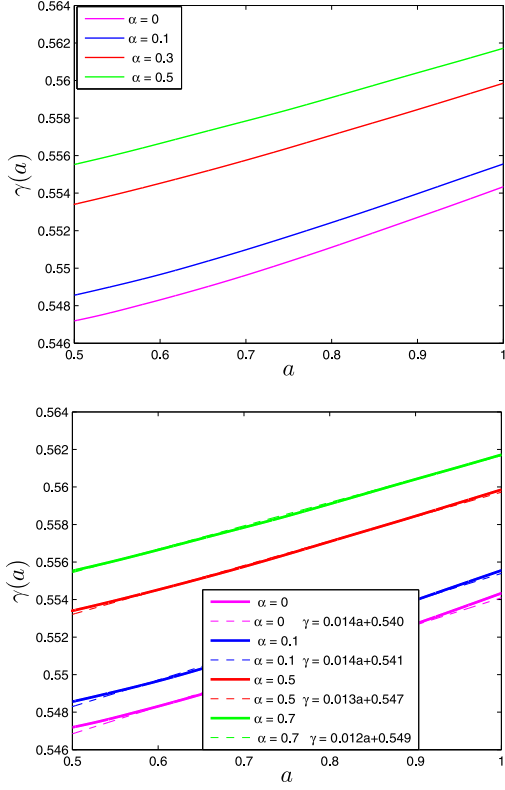


Fig. 8. 8a (upper panel) the $\gamma(a)$ function for the different values of the α parameter in the range of the redshifts $z \in (0; 1)$. 8b (lower panel) the $\gamma(a)$ function for the different values of the α parameter along with predictions, computed from the linear parameterizations.

parameter $w(a)$ in the Ratra-Peebles ϕ CDM model in the range of the redshifts $z \in (0; 1)$. While the Gerke-Estathiou and Chavalier-Polarsky-Linder parameterizations fit well the equation of state parameter $w(a)$ for the Ratra-Peebles ϕ CDM model on the large redshifts.

The expansion rates calculated for all aforementioned parameterizations of $w(a)$ fit well the expansion rates for the Ratra-Peebles ϕ CDM model for all values of the α parameter in the range of the redshifts $z \in (0; 0.6)$.

We've explored the Linder γ -parameterization for the Ratra-Peebles ϕ CDM and the Λ CDM models. The Linder γ -parameterization works well for both models and it is applicable in the range of the redshifts $z \in (0; 5)$.

We've found that the effective $\gamma(a)$ function can be approximated as a linear function in the range of the redshifts $z \in (0; 5)$, which coincides with the range of the redshifts applicability of the Linder γ -parameterization.

We've explored the observable predictions of the scalar field model. We've used a compilation of the BAO, the growth rate and the distance prior from the CMB to constrain the model parameters of the scalar field model. When only the growth rate data is applied, there is a strong degeneracy between the Ω_m and the α parameters.

Adding the BAO data and the distance prior from the CMB broke the degeneracy resulting in $\Omega_m = 0.3$ and $\alpha \leq 1.3$ with the best fit of $\alpha = 0$.

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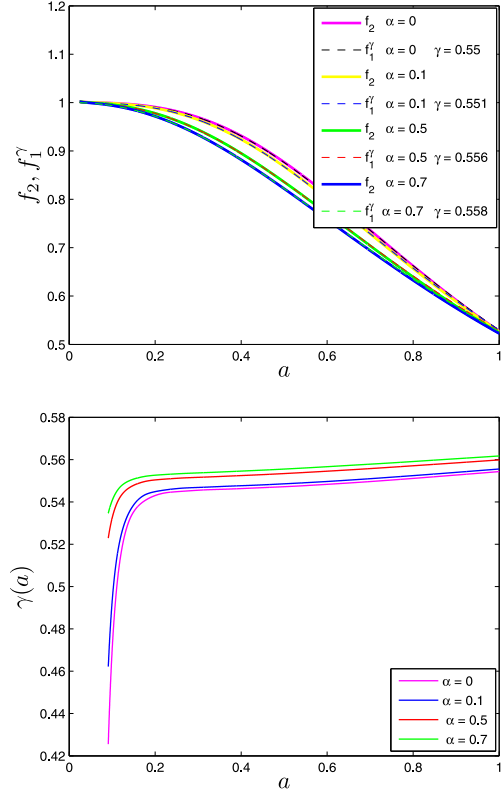


Fig. 9. 9a (upper panel) the logarithmic growth rate as a function of the scale factor f_2 for the different values of the α parameter (solid lines) along with the predictions f_1^γ (dashed lines), computed for the corresponding best-fit values of the γ parameter in the range of redshifts $z \in (0; 1)$. 9b (lower panel) the $\gamma(a)$ function for different values of the α parameter in the range of the redshifts $z \in (0; 10)$.

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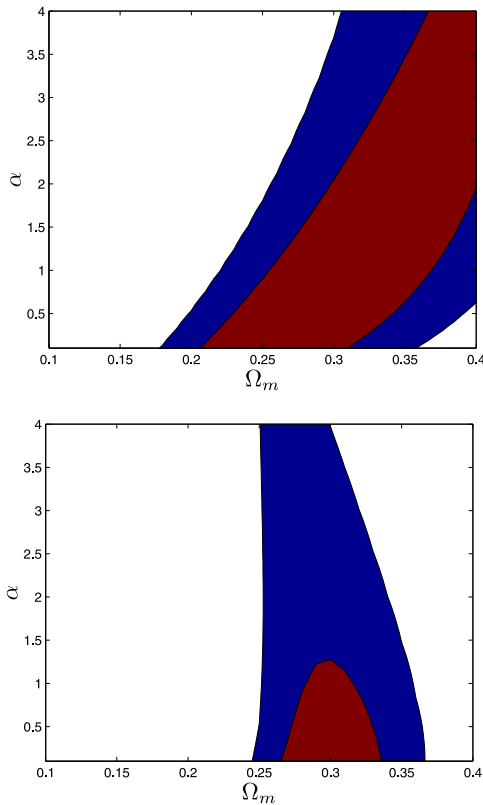


Fig. 10. 1σ and 2σ confidence level contours on the parameters Ω_m and α for the Ratra-Peebles ϕ CDM model. 10a (upper panel) the constraints, obtained from the growth rate data (Gupta et al., 2012). 10b (lower panel) the constraints, obtained after adding the BAO measurements and the cosmic microwave background (CMB) distance prior as in (Giotri et al., 2012) for the BAO/CMB distance prior.

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